Analytical solution for photorefractive screening solitons

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We study formation and interaction of one-dimensional screening solitons in a photorefractive medium with sublinear dependence of the photoconductivity on light intensity. We find an exact analytical solution to the corresponding nonlinear Schrödinger equation. We show that these solitons are stable in propagation and their interaction is generic for solitons of saturable nonlinearity. In particular, they may fuse or ''give birth'' to new solitons upon collision.

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The subject of screening spatial optical solitons in photorefractive crystals has attracted great interest in recent years. Screening solitons can be created using a very low optical power of the order of microwatts [1–4]. They are formed when the presence of an optical beam results in the screening of the external biasing dc field by a space charge field induced by distribution of photoexcited charge carriers. The resulting effective electric field modulates the refractive index of the medium via the Pockels effect. In effect, refractive index increases in the region of the beam leading to self-trapping and subsequent soliton propagation. The formation of one- and two-dimensional screening solitons has been demonstrated in a few photorefractive crystals including barium titanium oxide (BTO) and strontium barium niobate (SBN). Further, soliton interactions have also been studied and such effects as soliton spiraling, fusion, birth, and annihilation have been observed [5–10]. All these studies have been performed in bulk photorefractive crystals. Recently it has been demonstrated that screening solitons can also be formed in planar optical waveguides [11]. In this particular case, the waveguide was formed by an ion implantation into a SBN photorefractive crystal. By applying an electric field of 2.6 kV/cm spatial solitary beams of width of 8 μm have been observed. The same group also demonstrated experimentally the interaction of solitons in the planar waveguide [12].

An interesting aspect of these works is the fact that the SBN waveguide exhibits different photoconductive properties than does the bulk crystal. It is known that in the bulk crystal the photoconductivity is proportional to the light intensity. Consequently, the theoretical model governing soliton propagation corresponds to that of typical saturable nonlinearity (with the nonlinear term inversely proportional to the light intensity) [1,2,4]. On the other hand, the photoconductivity in the planar waveguide was found to depend on the square root of the light intensity [11,12]. This sublinear dependence will affect the profile of the generated solitons as well as their interaction.

In this work we will analyze the theoretical model for formation of solitons assuming the photoconductivity model as reported for SBN planar waveguides. We will show that the corresponding propagation equation has an exact analytical solution. We will use this solution to study the stability and interaction of the solitons. We will also illustrate the effect of a diffusive contribution to the nonlinearity on the trajectory of the soliton beam.

We consider the propagation of a one-dimensional optical beam with an amplitude $\tilde{u}(x,z)\exp(ik_0z)$ in an externally biased photorefractive medium, where k_0 is a wave vector. We assume that the beam propagates along the z axis and the biasing field is applied along the x axis. An additional uniform, broad beam provides the background illumination. The formation of the space charge field (with an amplitude $\vec{E}$) can be phenomenologically described by the set of equations [13]

$$\vec{j}=\sigma\vec{E}, \tag{1a}$$

$$\mathrm{div}\,\vec{j}=0, \tag{1b}$$

where $\vec{j}$ is the current density and σ is the photoconductivity. Now, following the experimental evidence of Ref. [12], we assume that the photoconductivity of the crystal varies with illumination as

$$\sigma\propto\sqrt{I+I_0}, \tag{2}$$

where $I=|\tilde{u}(x)|^2$ and I_0 are the beam and background light intensities, respectively. Then, assuming that all quantities depend on a single transverse coordinate (x) we get from Eq. (1b) that the current density is constant, $j(x)=\mathrm{const}=j_0$. Using this relation in Eq. (1a) we find that the electric field depends on the light intensity as

$$E(x)=E_0\frac{\sqrt{I_0}}{\sqrt{I+I_0}}, \tag{3}$$

with E_0 denoting the applied field in the absence of the soliton beam. This electric field modulates the refractive index of the crystal via the electro-optic effect $\Delta n=r_{eff}E(x)$, where r_{eff} is the effective electro-optic coefficient. Substituting Δn into the wave equation we obtain, in the slowly vary-

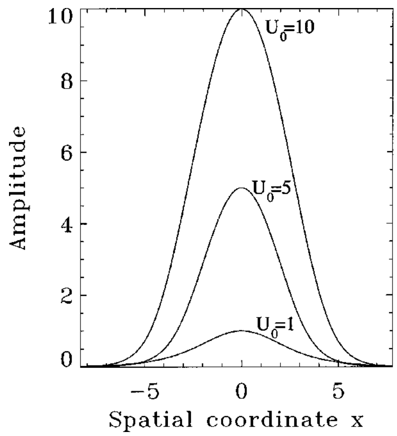

FIG. 1. Soliton profile for various levels of saturation.

ing envelope approximation, the following dimensionless equations for the beam amplitude $u(x)$:

$$i\frac{\partial u}{\partial z}+\frac{1}{2}\frac{\partial^2 u}{\partial x^2}-\frac{u}{\sqrt{|u|^2+1}}=0, \tag{4}$$

where the beam's amplitude has been normalized to background intensity $[u(x,z)=\tilde{u}(x,z)/\sqrt{I_0}]$. It is clear that the nonlinear term responsible for the self-focusing is of saturable character. For small beam intensity the nonlinearity is just that of Kerr material, while it saturates for high intensity. Interestingly, exactly the same propagation equation has been derived earlier by Segev *et al.* [14] using a perturbative approach to screening solitons in a SBN crystal in the case of high light intensity. Also, a similar equation has been studied in the context of soliton propagation in plasma [15].

We will be looking for the stationary soliton solution to Eq. (4) in the form

$$u(x,z)=u(x)\exp(i\Gamma z), \tag{5}$$

where Γ is the propagation constant. After substituting Eq. (5) into Eq. (4) and integrating once we find that the amplitude $u(x)$ satisfies the following first-order differential equation:

$$\frac{du}{dx}=\pm(2\Gamma u^2+4\sqrt{1+u^2}-4)^{1/2}, \tag{6}$$

where the propagation constant Γ is given by

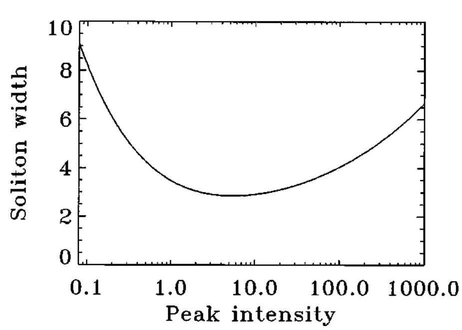

FIG. 2. Soliton width as a function of peak intensity.

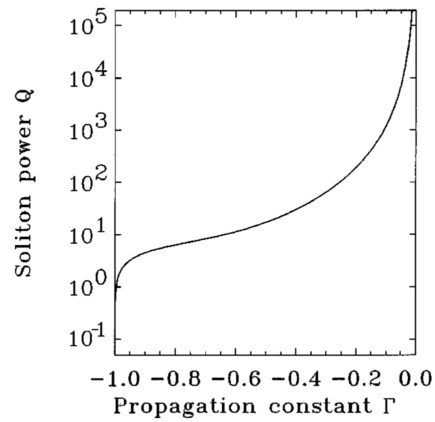

FIG. 3. Soliton power (Q) as a function of propagation constant (Γ). Its monotonic growth indicates linear stability of solitons.

$$\Gamma=-2\frac{\sqrt{1+u_0^2}-1}{u_0^2}, \tag{7}$$

with u_0 being the peak amplitude of the soliton. It turns out that Eq. (6) can be integrated and as a result one obtains

$$x=\frac{2}{\sqrt{2|\Gamma|}}\left(\sin^{-1}\frac{\sqrt{1+\sqrt{1+u^2}}\sqrt{2|\Gamma|}}{2}-\pi/2\right)-\frac{1}{\sqrt{8(1-|\Gamma|)}}\ln\left(\frac{2\sqrt{\mu_3 w}+\mu_2 y+2\mu_3}{\mu_2 y_0+2\mu_3}\frac{y_0}{y}\right), \tag{8}$$

where we introduced

$$\begin{aligned} y&=\sqrt{1+u^2}-1,\\ y_0&=\sqrt{1+u_0^2}-1,\\ w&=\mu_1 y^2+\mu_2 y+\mu_3,\\ \mu_1&=-2|\Gamma|,\\ \mu_2&=4(1-2|\Gamma|),\\ \mu_3&=8(1-|\Gamma|). \end{aligned} \tag{9}$$

Therefore Eq. (8) gives an implicit relation between the soliton amplitude and the spatial coordinate. In the case of low light intensity, i.e., for $|u_0^2|\ll 1$, we obtain from Eq. (8) the well-known sech profile for the Kerr soliton. In Fig. 1 we show a few examples of the soliton intensity profile for various degrees of saturation (determined by the soliton's peak intensity). It is evident that with increasing intensity the soliton becomes wider. This is because the saturation-induced weakening nonlinearity can only support a low-diffraction wider soliton. From the solution Eq. (8) one can obtain an explicit relation between the soliton full width at half maximum (FWHM) x_0 and its peak intensity. This relation is plotted in Fig. 2. It reflects behavior typical for a saturable

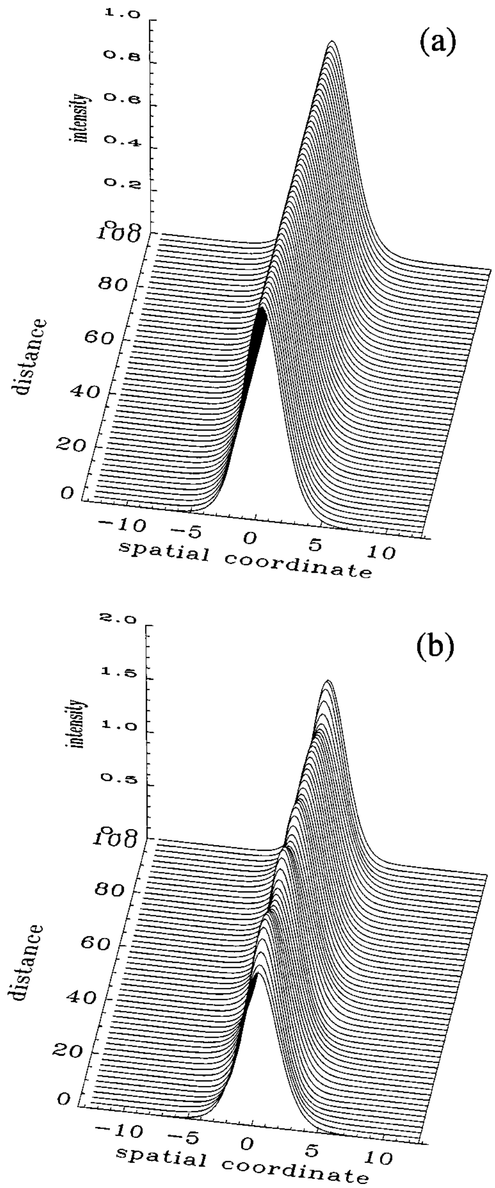

FIG. 4. Stability properties of the solitons Eq. (8); (a) propagation of an exact soliton solution; (b) propagation of the initially perturbed exact soliton solution.

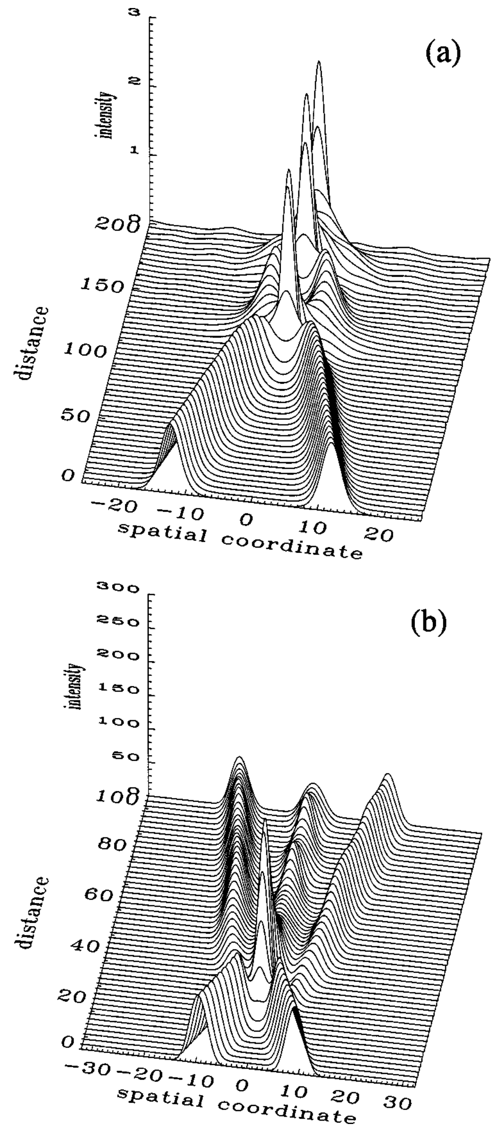

FIG. 5. Collision of (a) low- and (b) high-intensity spatial solitons; (a) soliton fusion; (b) birth of soliton.

nonlinearity. For small beam amplitudes the nonlinearity responds in a Kerr-type fashion and the soliton width decreases with intensity. For high intensity the nonlinearity saturates and the soliton width increases in order to minimize the effect of diffraction. As a result, there exist two soliton solutions (high- and low-intensity solitons) having exactly the same width. This property, known as ‘‘soliton bistability,’’ is generic for saturable nonlinearity [16].

An important aspect of any soliton solution is its stability. It is well known that in the case of fundamental solitons the stability properties can be determined from the dependence of the soliton power $Q=\int_{-\infty}^{\infty}u^2(x)dx$ on the propagation constant Γ. Solutions are stable (unstable) if $dQ/d\Gamma>0$ (<0) [20,21]. Using Eq. (6) one can find explicitly the relation $Q(\Gamma)$. This relation is plotted in Fig. 3. Clearly Q monotonically increases with Γ, which is an indication of the stability of the solitons. To demonstrate this stability we numerically integrated Eq. (4) using exact solutions as an initial condition. In Fig. 4 we show propagation of the individual soliton. The graph in Fig. 4(a) illustrates unperturbed propagation while that in Fig. 4(b) corresponds to the case when the initial soliton was perturbed by increasing its amplitude by 20%. It is evident that solitons are indeed stable.

Of great importance, from the practical point of view, are the collisional properties of solitons. It is well known that solitons described by integrable models are robust and collide elastically, preserving their shape and structure. On the other hand, in the case of nonintegrable models, soliton interaction is, in general, inelastic. As the saturable nonlinearity model discussed here is nonintegrable, the outcome of the soliton collision depends critically on the degree of saturation and intersection angle. For large interaction angles the collision is almost elastic and both solitons preserve their intensity profiles, experiencing only lateral shifts. We are dealing with a different scenario for smaller interaction angles. If the soliton intensity is low then the beams collide almost elastically (as Kerr-medium solitons do). For large saturation (i.e., high peak intensity) the collision becomes inelastic. Not only is the radiation always emitted from the impact area but also the outcome of the collision strongly depends on the relative phase of the solitons [17,18]. In particular, two solitons colliding in phase at small intersecting angle may merge, forming a high-intensity broad soliton. This situation is displayed in Fig. 5(a). On the other hand, increasing the intersecting angle slightly leads to formation of an additional soliton. This is the so-called ''soliton birth'' [18,19] that has been recently observed in experiments with bulk SBN photorefractive crystal [8] as well as a planar waveguide [12]. Figure 5(b) shows just that. Two incoming high-intensity spatial solitons intersect and as a result of beam interaction, three new solitary beams emerge from the impact area. This phenomenon can be used to construct passive multiport optical switches.

So far in our discussion, the nonlinearity that is responsible for formation of screening solitons was of local nature. However, it is well known that the photorefractive effect leads also to a nonlocal contribution to the refractive index change. This contribution, caused by diffusion of photoexcited charge carriers, leads to change of the beam's trajectory, the so-called self-bending effect [22,23]. Its role becomes important for very narrow optical beams. It can be shown that the self-bending effect can be taken into account by adding to the left-hand side of the propagation equation (4) an additional term

$$\gamma \frac{1}{(1+|u|^2)} \frac{\partial |u|^2}{\partial x}, \tag{10}$$

where the parameter γ determines the relative strength of the diffusive contribution to the total refractive index change. To show the effect of this term on soliton propagation we integrated the modified propagation equation using an exact soliton solution as an initial condition. The result of integration is shown in Fig. 6 where the self-bending of the soliton is clearly visible.

In conclusion, we investigated properties of one-dimensional spatial solitons in photorefractive material with a sublinear dependence of photoconductivity on light intensity. This type of photoconductivity has been observed in thin films of strontium barium niobate crystals. It results in a saturable character of the photorefractive nonlinearity with the light-induced refractive index change being inversely proportional to the square root of the light intensity.

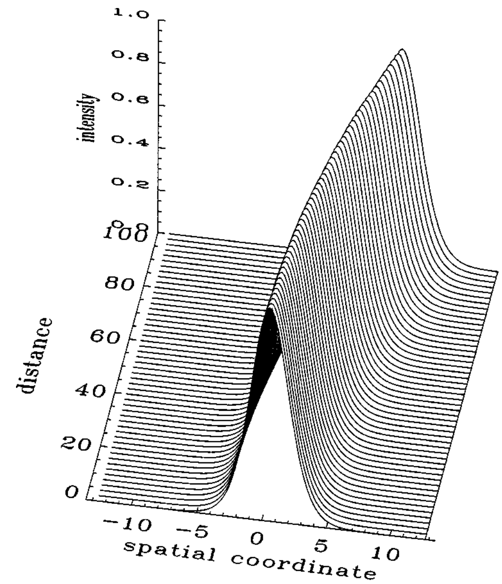

FIG. 6. Diffusion-induced self-bending of the screening soliton; parameter $\gamma = 0.01$.

We found exact analytical soliton solutions to the corresponding nonlinear Schrödinger equations. We showed that these solitons are stable in propagation. Their collisions are in general inelastic and typical for those described by other nonintegrable models. In particular, they may result in soliton fusion or formation of new solitons. However, because of weaker dependence on light intensity than, e.g., in the case of standard screening solitons, here the inelastic character of collisions becomes evident at higher light intensities. We also showed that the presence of diffusion of photoexcited charges results in self-bending of the solitons.

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